**BINOMIAL TREE: A TOOL FOR OPTION PRICING**

There are several tools for option pricing, of which binomial trees are one of the most basic. In this article we develop the idea behind it and some algebra.

*1.1 A counter-intuitive beginning on option value*

The binomial tree is based on the assumption that we can model the price of a stock in two different states: up and down, with a certain probability. Let’s assume that from an initial price of 100 the stock can go to 101 or 99, with p=0.7 and p’=0.3 respectively. We introduce a call option on the stock, asking ourselves the value of the option. We would be inclined to say 0.7 taking the expectation (0.7\*(101-100)), but we would be wrong.. In order to get the option value we use the concept of no arbitrage: there is not risk-free money to be made in our market. Now consider two different portfolios:

A). Long one option and short half (we’ll later where this derives from) stock;

B). Risk-free security

With portfolio A, in the up scenario, the payoff would be (101-100)-0.5\*100=-99/2; in the down scenario we would have 0-99/2=-99/2.

We have therefore constructed a risk free portfolio and assuming the interest rate are 0, the value (payoff) tomorrow must be the same as the ones from today.

Therefore, at time t=0, we have: option value – 100/2 = -99/2, finding the option value is ½.

What tricked us in the first place was the probability of the stock having an up movement: the value of an option does not depend on the probability of raising and falling. This is since we have hedged the option with a stock and therefore don’t care whether the stocks rises or falls. What we care about is the stock’s **volatility.**

½ is chosen as to hedge the portfolio, in general

*1.2 And the role of risk in pricing stocks..*

The expected stock value tomorrow is 0.7\*101+0.3\*99 = 100.4. Why is it not the price of the stock today? Because the stock investment is risky and therefore a positive expected return is expected.

*1.3 The real and risk-neutral worlds*

The case we have presented is based in the **real-world** where we are sensitive to risk, expecting greater returns for taking risk. Another setting is the **risk-neutral world**, where risk is not taken into consideration and we price using expectations.

In this setting p(RN) and p’(RN) would be 0.5: the stock is currently priced at 100 today and the possible values are 101 and 99 in the future. p(RN)\*101+p’(RN)\*99 =100. These probabilities assumes that expectation are used for pricing and no allowance has been made for risk.

*The model (Cox-Ross-Rubenstein)*

Single-period replication: consider the following stock with its stock price evolution and the risk free rate is R =

100

50

25

Now consider a call with strike K=25.

50

C=?

0

Considering a long position of in the stock and dollars in the bond, we have two different payoffs:

UP:

DOWN:

Defining the payoff of the call in the UP as and in the DOWN case as we have that:

;

(Same as before where we considered Call – stock, but here there is interest rate); solving we have

;

The formula for Delta is the same in paragraph 1.1

The value of the call is therefore:

Which substituting gives us

Defining and we have

is the probability of the stock going up in a risk-neutral world.

**References: Paul Wilmott on Quantitative Finance, cap. 3**